$$\textbf{Eliminate}\left[\left\{a == \sqrt{(xr - x)^2 + y^2} + \sqrt{x^2 + y^2},\ b == \sqrt{(xl - x)^2 + y^2} + \sqrt{x^2 + y^2}\right\}, x\right]$$

$$b^4 \left(a^2 - xr^2\right) + a\, b^3 \left(-2 a^2 + 2 xr^2\right) + a\, b\, xl \left(2 a^2 xl - 2 xl\, xr^2 - 8 xr\, y^2\right) + b^2 \left(a^4 - 2 a^2 xl^2 + 2 a^2 xl\, xr - 2 a^2 xr^2 + 2 xl^2 xr^2 - 2 xl\, xr^3 + xr^4 + 4 xr^2 y^2\right) == xl^2 \left(a^4 - a^2 xl^2 + 2 a^2 xl\, xr - 2 a^2 xr^2 + xl^2 xr^2 - 2 xl\, xr^3 + xr^4 - 4 a^2 y^2\right)$$

$$\textbf{Solve}\left[b^4 \left(a^2 - xr^2\right) + a\, b^3 \left(-2 a^2 + 2 xr^2\right) + a\, b\, xl \left(2 a^2 xl - 2 xl\, xr^2 - 8 xr\, y^2\right) + b^2 \left(a^4 - 2 a^2 xl^2 + 2 a^2 xl\, xr - 2 a^2 xr^2 + 2 xl^2 xr^2 - 2 xl\, xr^3 + xr^4 + 4 xr^2 y^2\right) == xl^2 \left(a^4 - a^2 xl^2 + 2 a^2 xl\, xr - 2 a^2 xr^2 + xl^2 xr^2 - 2 xl\, xr^3 + xr^4 - 4 a^2 y^2\right), y\right]$$

$$\Big\{\Big\{y \to -\frac{1}{2\sqrt{a^2 xl^2 - 2 a\, b\, xl\, xr + b^2 xr^2}} \Big(\sqrt{\big(-a^4 b^2 + 2 a^3 b^3 - a^2 b^4 + a^4 xl^2 - 2 a^3 b\, xl^2 + 2 a^2 b^2 xl^2 - a^2 xl^4 - 2 a^2 b^2 xl\, xr + 2 a^2 xl^3 xr + 2 a^2 b^2 xr^2 - 2 a\, b^3 xr^2 + b^4 xr^2 - 2 a^2 xl^2 xr^2 + 2 a\, b\, xl^2 xr^2 - 2 b^2 xl^2 xr^2 + xl^4 xr^2 + 2 b^2 xl\, xr^3 - 2 xl^3 xr^3 - b^2 xr^4 + xl^2 xr^4\big)}\Big)\Big\},$$

$$\Big\{y \to \frac{1}{2\sqrt{a^2 xl^2 - 2 a\, b\, xl\, xr + b^2 xr^2}} \Big(\sqrt{\big(-a^4 b^2 + 2 a^3 b^3 - a^2 b^4 + a^4 xl^2 - 2 a^3 b\, xl^2 + 2 a^2 b^2 xl^2 - a^2 xl^4 - 2 a^2 b^2 xl\, xr + 2 a^2 xl^3 xr + 2 a^2 b^2 xr^2 - 2 a\, b^3 xr^2 + b^4 xr^2 - 2 a^2 xl^2 xr^2 + 2 a\, b\, xl^2 xr^2 - 2 b^2 xl^2 xr^2 + xl^4 xr^2 + 2 b^2 xl\, xr^3 - 2 xl^3 xr^3 - b^2 xr^4 + xl^2 xr^4\big)}\Big)\Big\}\Big\}$$

$$\textbf{FullSimplify}\Big[\Big\{\Big\{y \to -\frac{1}{2\sqrt{a^2 xl^2 - 2 a\, b\, xl\, xr + b^2 xr^2}} \Big(\sqrt{\big(-a^4 b^2 + 2 a^3 b^3 - a^2 b^4 + a^4 xl^2 - 2 a^3 b\, xl^2 + 2 a^2 b^2 xl^2 - a^2 xl^4 - 2 a^2 b^2 xl\, xr + 2 a^2 xl^3 xr + 2 a^2 b^2 xr^2 - 2 a\, b^3 xr^2 + b^4 xr^2 - 2 a^2 xl^2 xr^2 + 2 a\, b\, xl^2 xr^2 - 2 b^2 xl^2 xr^2 + xl^4 xr^2 + 2 b^2 xl\, xr^3 - 2 xl^3 xr^3 - b^2 xr^4 + xl^2 xr^4\big)}\Big)\Big\},$$

$$\Big\{y \to \frac{1}{2\sqrt{a^2 xl^2 - 2 a\, b\, xl\, xr + b^2 xr^2}} \Big(\sqrt{\big(-a^4 b^2 + 2 a^3 b^3 - a^2 b^4 + a^4 xl^2 - 2 a^3 b\, xl^2 + 2 a^2 b^2 xl^2 - a^2 xl^4 - 2 a^2 b^2 xl\, xr + 2 a^2 xl^3 xr + 2 a^2 b^2 xr^2 - 2 a\, b^3 xr^2 + b^4 xr^2 - 2 a^2 xl^2 xr^2 + 2 a\, b\, xl^2 xr^2 - 2 b^2 xl^2 xr^2 + xl^4 xr^2 + 2 b^2 xl\, xr^3 - 2 xl^3 xr^3 - b^2 xr^4 + xl^2 xr^4\big)}\Big)\Big\}\Big\}\Big]$$

$$\left\{\left\{y \to -\frac{\sqrt{\left(-b^2 + xl^2\right)(a - xr)(a - b + xl - xr)(a + xr)(a - b - xl + xr)}}{2\sqrt{(a\, xl - b\, xr)^2}}\right\}, \left\{y \to \frac{\sqrt{\left(-b^2 + xl^2\right)(a - xr)(a - b + xl - xr)(a + xr)(a - b - xl + xr)}}{2\sqrt{(a\, xl - b\, xr)^2}}\right\}\right\}$$

$$\textbf{Eliminate}\left[\left\{a == \sqrt{(xr - x)^2 + y^2} + \sqrt{x^2 + y^2},\ b == \sqrt{(xl - x)^2 + y^2} + \sqrt{x^2 + y^2}\right\}, y\right]$$

$$a\, b^2 + b\left(-a^2 - 2 x\, xr + xr^2\right) == a\, xl\,(-2 x + xl)$$

$$\text{Solve}\left[a\,b^2 + b\left(-a^2 - 2\,x\,xr + xr^2\right) == a\,xl\,(-2\,x + xl),\ x\right]$$

$$\left\{\left\{x \to \frac{a^2\,b - a\,b^2 + a\,xl^2 - b\,xr^2}{2\,(a\,xl - b\,xr)}\right\}\right\}$$

In[1]:= $$\text{FullSimplify}\left[\left\{\left\{x \to \frac{a^2\,b - a\,b^2 + a\,xl^2 - b\,xr^2}{2\,(a\,xl - b\,xr)}\right\}\right\}\right]$$

Out[1]= $$\left\{\left\{x \to \frac{a\left((a-b)\,b + xl^2\right) - b\,xr^2}{2\,a\,xl - 2\,b\,xr}\right\}\right\}$$

Vysledne vztahy pro výbočet bodu odrazu ve tvaru použtém v programu
f, g – jsou dráhy zm■■ené sonarem.

$$y == \sqrt{\frac{\left(-g^2 + xl^2\right)(f - xr)(f - g + xl - xr)(f + xr)(f - g - xl + xr)}{4\,(f\,xl - g\,xr)^2}}\ ,\quad x == \frac{f\left((f-g)\,g + xl^2\right) - b\,xr^2}{2\,f\,xl - 2\,g\,xr}$$

Vyjadreni vztahu pro kontrolovanou trojúhelníkovou nerovnost.

In[8]:= $$\text{Eliminate}\left[\left\{c == \sqrt{x^2 + y^2},\ a == \sqrt{(x - xl)^2 + y^2},\ b == \sqrt{(xr - x)^2 + y^2},\ a + c == f,\ b + c == g\right\},\ \{x, y, a, b\}\right] ■$$

Out[8]= $$■\ \left(xl\left(-2\,c\,g + g^2 + xl\,xr - xr^2\right) == -2\,c\,f\,xr + f^2\,xr\right)$$

In[10]:= $$\text{Solve}\left[xl\left(-2\,c\,g + g^2 + xl\,xr - xr^2\right) == -2\,c\,f\,xr + f^2\,xr,\ c\right]$$

Out[10]= $$\left\{\left\{c \to \frac{g^2\,xl - f^2\,xr + xl^2\,xr - xl\,xr^2}{2\,(g\,xl - f\,xr)}\right\}\right\}$$

In[18]:= $$\text{FullSimplify}\left[\frac{g^2\,xl - f^2\,xr + xl^2\,xr - xl\,xr^2}{2\,(g\,xl - f\,xr)} == c\right]$$

Out[18]= $$\frac{-g^2\,xl + xr\left(f^2 + xl\,(-xl + xr)\right)}{-2\,g\,xl + 2\,f\,xr} == c$$

In[12]:= $$\text{Eliminate}\left[\left\{c == \sqrt{x^2 + y^2},\ a == \sqrt{(x - xl)^2 + y^2},\ b == \sqrt{(xr - x)^2 + y^2},\ a + c == f,\ b + c == g\right\},\ \{x, y, a, c\}\right] ■$$

Out[12]= $$■\ \left(xl\left(2\,b\,g - g^2 + xl\,xr - xr^2\right) == f^2\,xr + f\,(2\,b - 2\,g)\,xr\right)$$

In[14]:= $$\text{Solve}\left[xl\left(2\,b\,g - g^2 + xl\,xr - xr^2\right) == f^2\,xr + f\,(2\,b - 2\,g)\,xr,\ b\right]$$

In[23]:= $\left\{\left\{b \to \frac{g^2\, xl + f^2\, xr - 2\, f\, g\, xr - xl^2\, xr + xl\, xr^2}{2\,(g\, xl - f\, xr)}\right\}\right\}$

$\text{FullSimplify}\left[b == \frac{g^2\, xl + f^2\, xr - 2\, f\, g\, xr - xl^2\, xr + xl\, xr^2}{2\,(g\, xl - f\, xr)}\right]$

Out[23]= $\left\{\left\{b \to \frac{g^2\, xl + f^2\, xr - 2\, f\, g\, xr - xl^2\, xr + xl\, xr^2}{2\,(g\, xl - f\, xr)}\right\}\right\}$

Out[24]= $b == \frac{g^2\, xl - 2\, f\, g\, xr + xr\,\left(f^2 + xl\,(-xl + xr)\right)}{2\, g\, xl - 2\, f\, xr}$

In[16]:= Eliminate[

$\left\{c == \sqrt{x^2 + y^2},\ a == \sqrt{(x - xl)^2 + y^2},\ b == \sqrt{(xr - x)^2 + y^2},\ a + c == f,\ b + c == g\right\},\ \{x, y, b, c\}]$

Out[16]= $f^2\, xr + f\,(-2\, g\, xl - 2\, a\, xr) == xl\,\left(-2\, a\, g - g^2 - xl\, xr + xr^2\right)$

In[17]:= $\text{Solve}\left[f^2\, xr + f\,(-2\, g\, xl - 2\, a\, xr) == xl\,\left(-2\, a\, g - g^2 - xl\, xr + xr^2\right),\ a\right]$

Out[17]= $\left\{\left\{a \to \frac{2\, f\, g\, xl - g^2\, xl - f^2\, xr - xl^2\, xr + xl\, xr^2}{2\,(g\, xl - f\, xr)}\right\}\right\}$

In[25]:= $\text{FullSimplify}\left[\left\{\left\{a \to \frac{2\, f\, g\, xl - g^2\, xl - f^2\, xr - xl^2\, xr + xl\, xr^2}{2\,(g\, xl - f\, xr)}\right\}\right\}\right]$

Out[25]= $\left\{\left\{a \to \frac{-2\, f\, g\, xl + f^2\, xr + xl\,\left(g^2 + (xl - xr)\, xr\right)}{-2\, g\, xl + 2\, f\, xr}\right\}\right\}$

Vysledne strany trojuhelnik■ nutné pro zjist■ní existence výsledného bodu.

$$a == \frac{-2\, f\, g\, xl + f^2\, xr + xl\,\left(g^2 + (xl - xr)\, xr\right)}{-2\, g\, xl + 2\, f\, xr},$$

$$b == \frac{g^2\, xl - 2\, f\, g\, xr + xr\,\left(f^2 + xl\,(-xl + xr)\right)}{2\, g\, xl - 2\, f\, xr},\ c == \frac{-g^2\, xl + xr\,\left(f^2 + xl\,(-xl + xr)\right)}{-2\, g\, xl + 2\, f\, xr}$$