Příklady k přednášce 8
Exercises for Lecture 8

1. Given:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & -4 & -6 & -4 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} c_1 & c_2 & 1 & 0 \end{bmatrix}.$$

   (a) Show that the set of values of (c_1, c_2) such that the realization is not minimal, is a line on the (c_1, c_2) plane.

   (b) Is $(c_1, c_2) = (2, 3)$ on that line? If so, use Kalman decomposition to find a minimal realization.

   (c) A friend from VUT Brno told you that there exists a second order realization of this system for some (c_1, c_2). Is it possible? If yes, find those values for c_1 and c_2.

2. Consider a system described by

$$H(s) = \begin{bmatrix} \frac{1}{s+1} & \frac{2}{s+1} \\ \frac{-1}{(s+1)(s+2)} & \frac{1}{s+2} \end{bmatrix}.$$

   (a) What is the order of a controllable and observable realization of this system?

   (b) If we consider such a realization, is the resulting system controllable from the 1st input only? Is it controllable from the 2nd input only?

   (c) And still with the same realization. Is it observable from the 1st output only? Is it observable from the 2nd output only?

3. Consider a system giving rise to the transfer function matrix

$$H(s) = \begin{bmatrix} \frac{1}{(s-1)^2} & \frac{1}{(s-1)(s+3)} \\ \frac{-6}{(s-1)(s+3)^2} & \frac{s-2}{(s+3)^2} \end{bmatrix}.$$

   (a) Determine a realization of $H(s)$ in controller form.

   (b) What is the order of minimal realization of $H(s)$?

4. Consider the system

$$H(s) = \begin{bmatrix} \frac{4s^2-s-1}{s^3-s} & \frac{s+3}{s^2-1} \\ \frac{-s^2+s-2}{s^3-s} & \frac{1}{s+1} \end{bmatrix}$$

   (a) Find a realization with A diagonal

   (b) Show whether the realization is minimal. If it is not, find a minimal realization.

5. Determine

   (a) an observable realization,

   (b) a balanced minimal realization

   of the transfer function

$$H(s) = \frac{s^3 + s^2}{s^3 + s^2 + s + 1}$$