Příklady k přednášce 4
Exercises for Lecture 4

1. Consider the scalar system $\dot{x} = x + u$. Derive an input that will drive the state $x_0 = 0$ at $t = 0$ to $x_1 = \alpha$ in T seconds. Does this input maintain the state at α?

2. Consider the state equation

$$x(k+1) = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} x(k) + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} u(k).$$

   Is the state

$$x^1 = \begin{bmatrix} 1 \\ 3 \\ 3 \end{bmatrix}$$

   reachable? If yes, what is the minimum number of steps required to transfer the state from the zero state to x^1? What inputs do you need? What is the set of all states that are reachable?

3. Determine the uncontrollable modes of the pair (A, B) given by

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

   using eigenvalue/eigenvector criteria.

4. Consider the discrete-time system that is obtained by sampling the continuous-time system

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u,$$

   with sampling period T. Determine the values of T that preserve controllability.

5. Consider the state equation

$$x(k+1) = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} x(k) + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} u(k).$$

   Determine all states that are controllable.