Příklady k přednášce 6
Exercises for Lecture 6

1. Reduce the pair

$$A=\begin{bmatrix}0&0&1&0\\3&0&-3&1\\-1&1&4&-1\\1&0&-1&0\end{bmatrix},\quad B=\begin{bmatrix}0&0\\1&1\\0&0\\0&1\end{bmatrix}$$

into controller form $A_c = PAP^{-1}$, $B_c = PB$. What is the similarity transformation matrix P in this case? What are the controllability indices?

2. Consider a system (A, B) with a diagonal matrix A. Using the eigenvalue test, determine a condition for (A, B) to be controllable.

3. Determine the unobservable modes of the pair (A, C) given by

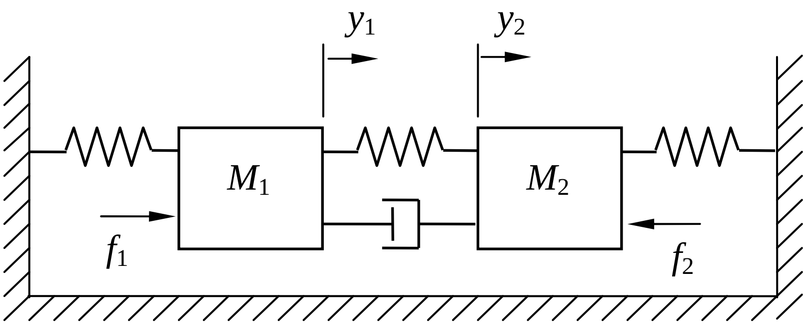
$$A=\begin{bmatrix}0&0&0&0\\0&0&0&0\\0&1&0&0\\0&0&0&-1\end{bmatrix},\quad C=\begin{bmatrix}0&0&1&0\\1&0&0&0\end{bmatrix}.$$

4. Consider the system (A, B, C, D) given by

$$A=\begin{bmatrix}0&0&0\\1&-2&1\\0&0&0\end{bmatrix},\quad B=\begin{bmatrix}1&0\\1&1\\1&2\end{bmatrix},\quad C=\begin{bmatrix}0&1&0\end{bmatrix},\quad D=\begin{bmatrix}1&0\end{bmatrix}.$$

Reduce the system to Kalman's canonical form.

5. Consider a spring mass system, which consists of two point masses that are acted upon by viscous damping forces, spring forces and external forces f_1 and f_2. The displacements of the masses are denoted by y_1 and y_2.



Letting $x_1 = y_1, x_2 = \dot{y}_1$ and $x_3 = y_2, x_4 = \dot{y}_2$, we can describe the motions of the masses (assuming particular values of the masses, the spring constants, and the viscous damping coefficients) by

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -0.1910 & -0.0536 & 0.0910 & 0.0036 \\ 0 & 0 & 0 & 1 \\ 0.0910 & 0.0036 & -0.1910 & -0.0536 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \end{bmatrix}.$$

Suppose that the displacement and the velocity of the first mass are measured,

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}.$$

(a) Is the system controllable?

(b) Is the system controllable from input f_1 only? Is it reachable from f_2 only?

(c) Is the system observable?

(d) Is the system observable from output y_1 only? Is it observable from y_2 only?

observability.