Příklady k přednášce 9
Exercises for Lecture 9

1. Consider the system

$$\dot{x} = \begin{bmatrix} -0.01 & 0 \\ 0 & -0.02 \end{bmatrix} x + \begin{bmatrix} 1 & 1 \\ -0.25 & 0.75 \end{bmatrix} u,$$

with $u = Fx$. Verify that the three different state feedback matrices given by

$$F_1 = \begin{bmatrix} -1.1 & -3.7 \\ 0 & 0 \end{bmatrix}, \quad F_2 = \begin{bmatrix} 0 & 0 \\ -1.1 & 1.2333 \end{bmatrix}, \quad F_3 = \begin{bmatrix} -0.1 & 0 \\ 0 & -0.1 \end{bmatrix}$$

all assign the closed-loop eigenvalues at the same locations, namely, at $-0.1025 \pm j0.04944$. For all the three cases plot $x(t) = [x_1(t) \quad x_2(t)]^T$ when $x_1(0) = 0$, $x_2(0) = 1$. This example demonstrates how different the responses can be for different designs even though the eigenvalues of the compensated system are at the same locations.

2. For the system $\dot{x} = Ax + Bu$, where

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & -3 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix},$$

determine F so that the eigenvalues of $A + BF$ are at $-1 \pm j$ and $-2 \pm j$. Accomplish this by reducing the system to a single-input controllable system.

3. Consider the system $\dot{x} = Ax + Bu$, where

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

and the performance index

$$J = \int_0^\infty (x^T Q x + u^T R u) dt,$$

where

$$Q = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad R = 1.$$

Determine the optimal control law that minimizes J.

4. Consider the system $\dot{x} = Ax + Bu,\ y = Cx + Du$, where

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad C = [1 \quad 0], \quad D = 1.$$

Let $u = Fx + r$ be a linear state feedback control law. Determine F so that the eigenvalues of $A + BF$ are unobservable from y. What is the closed-loop transfer function $H_F(s)$, where $\hat{y} = H_F \hat{r}$, in this case?

A servomotor that drives a load is described by the equation $\ddot{\theta} + \dot{\theta} = u,$ where θ is the shaft position (output) and u is the applied voltage. Choose u so that θ and $\dot{\theta}$ will go to zero exponentially. To accomplish this, derive a state-space representation of the servomotor and determine linear state feedback, $u = Fx + r$, so that both closed-loop eigenvalues are at –2.