$\text{Eliminate}\left[\left\{a^2 == y^2 + x^2,\ b^2 == y^2 + (r - x)^2\right\}, x\right]$

$-b^4 + 2\,b^2\,r^2 - r^4 - 4\,r^2\,y^2 == a^4 + a^2\left(-2\,b^2 - 2\,r^2\right)$

In[1]:= $\text{Simplify}\left[-b^4 + 2\,b^2\,r^2 - r^4 - 4\,r^2\,y^2 == a^4 + a^2\left(-2\,b^2 - 2\,r^2\right)\right]$

Out[1]= $a^4 + b^4 + r^4 + 4\,r^2\,y^2 == 2\,b^2\,r^2 + 2\,a^2\left(b^2 + r^2\right)$

In[2]:= $\text{Solve}\left[a^4 + b^4 + r^4 + 4\,r^2\,y^2 == 2\,b^2\,r^2 + 2\,a^2\left(b^2 + r^2\right), y\right]$

Out[2]= $\left\{\left\{y \to -\dfrac{\sqrt{-a^4 + 2\,a^2\,b^2 - b^4 + 2\,a^2\,r^2 + 2\,b^2\,r^2 - r^4}}{2\,r}\right\},\ \left\{y \to \dfrac{\sqrt{-a^4 + 2\,a^2\,b^2 - b^4 + 2\,a^2\,r^2 + 2\,b^2\,r^2 - r^4}}{2\,r}\right\}\right\}$

In[3]:= $\text{Eliminate}\left[\left\{a^2 == y^2 + x^2,\ b^2 == y^2 + (r - x)^2\right\}, y\right]$

Out[3]= $b^2 - r^2 + 2\,r\,x == a^2$

In[4]:= $\text{Simplify}\left[b^2 - r^2 + 2\,r\,x == a^2\right]$

Out[4]= $a^2 + r\,(r - 2\,x) == b^2$

In[5]:= $\text{Solve}\left[a^2 + r\,(r - 2\,x) == b^2, x\right]$

Out[5]= $\left\{\left\{x \to \dfrac{a^2 - b^2 + r^2}{2\,r}\right\}\right\}$